Multilevel Optimization
Using Recursive Trust-Region Methods

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Abstract: Many large-scale finite-dimensional optimization problems arise from the discretization of infinite-dimensional problems, a primary example being optimal-control problems defined in terms of either ordinary or partial differential equations. While the direct solution of such problems for a discretization level yielding the desired accuracy is often possible using existing packages for large-scale numerical optimization, this technique typically does make very little use of the fact that there is an underlying infinite-dimensional problem for which several discretization levels are possible, and the approach thus rapidly becomes cumbersome. This observation motivates the developments presented in this talk, where we explore the properties of a class of algorithms which, at variance with the technique just described, makes explicit use of the discretization in the hope to allow better efficiency and, possibly, enhance reliability.

Using the different possible levels of discretization for an infinite-dimensional problem is not a new idea. It is interesting that most of the existing proposals (including contributions by Gelman and Mandel (1990), Nash and Lewis (2000), Alexandrov and co-authors (2001), Kunish and Borzi (2005)) are all based on linesearch algorithms. The class of algorithms discussed in the talk can be viewed as an alternative where one uses the trust-region technology whose efficiency and reliability in the solution of nonconvex problems is well-known (see the SIAM book by Conn et al. (2000) for a more complete coverage of this subject). Our developments will be organized as follows. We will first describe our class of multiscale trust-region algorithms, and show that it contains a method that performs well on some examples. This observation then motivates an overview of the theory available so far for such methods. This includes results on the convergence of the generated iterates to first-order and weakly second-order critical points. Some conclusions and perspectives will finally be presented.

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