Numerical Treatment of Integro-PDEs for Phytoplankton Dynamics

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Abstract: Modelling the dynamics of phytoplankton is of great importance to many aspects of human interest, since phytoplankton provides the basis of the food chain in lakes, seas and oceans. A particularly interesting aspect is the ability of sinking phytoplankton species to take up CO_2 from the atmosphere, resulting in a downward export of carbon to the bottom of the ocean (the so-called 'biological pump'). By this mechanism, several gigatons per year of carbon dioxide are removed from the atmosphere, thus making a significant contribution to the reduction of the greenhouse problem on earth.

Phytoplankton requires light for photosynthesis. As a result, the production rate, which is determined by the local light intensity, decreases with depth, due to absorption. Furthermore, mortality rates and transport by turbulent diffusion in a water column (mixing) play a role. Also, phytoplankton species often have a specific weight different from that of water, giving rise to vertical transport in the form of sinking or buoyancy. Taking all these processes into account, leads to an **integro-partial differential equation of advection-diffusion-reaction type**.

Usually, light availability is the major factor limiting phytoplankton growth. In some regions, however, phytoplankton growth is limited by the availability of nutrients, such as nitrogen, iron, and phosphoros. We will consider a model in which *both limiting factors, light and nutrient*, are taken into account. These two factors give rise to contrasting gradients since light is coming from above, whereas nutrients are supplied at the sediment. As a result, the vertical distribution of the phytoplankton population can be quite heterogeneous in the sense that a large aggregation of phytoplankton is formed at a subsurface depth, where both light and nutrient are just sufficiently available to sustain a population. In a certain part of parameter space, it turns out that the biomass (as a function of time) shows an oscillatory behaviour. So far, nutrient limitation of phytoplankton is thought to lead to a stable equilibrium without oscillations.

The above aspect will be illustrated and the underlying algorithms in the numerical simulations will be discussed.

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