

# Adaptive Software for Solving Fourth-Order Time Dependent PDEs

R. D. Russell<sup>1</sup>, J. F. Williams<sup>2</sup>, and X. Xu<sup>3</sup>

**Abstract:** To gain insight into the dynamics of rich spatial and temporal patterns in a wide range of physical and mechanical problems, fourth-order model equations and systems of fourth-order equations have been a topic of much recent interest. Such equations arise, e.g., in thin film theory, lubrication theory, convection-explosion theory, flame and wave propagation, phase transition at critical Lipschitz points, and bi-stable systems.

An adaptive strategy based largely upon basic moving mesh methods is found to be particularly effective for solving many important fourth-order problems, especially because the scaling invariance of these problems can be incorporated. The resulting moving mesh code MOVCOL4 applies a collocation method directly to the fourth-order differential equations instead of converting the equations to lower order systems. Some properties of MOVCOL4 as well as alternative methods for solving fourth-order PDEs are briefly discussed and several problems solved to demonstrate the advantages of this approach and the breadth of its applicability.

---

<sup>1,2,3</sup> Department of Mathematics  
Simon Fraser University  
Burnaby, BC  
V5A 1S6 Canada  
*rdr@cs.sfu.ca, jfw@cs.sfu.ca, xxub@sfu.ca*