Solving Max-Cut Problems by Semidefinite Programming

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Abstract: We consider the Max-Cut problem in the form of quadratic programming in $-1, +1$ variables x_i:

$$\max x^T L x \text{ such that } x \in \{-1, 1\}^n.$$

Here L denotes the Laplacian of the underlying (weighted) graph. The resulting relaxation, obtained by the change of variables $X = xx^T$ leads to the following optimization problem in the semidefinite matrix variable X:

$$\max \langle L, X \rangle \text{ such that } X \succeq 0, \; x_{ii} = 1 \forall i.$$

This relaxation can be further tightened by adding some combinatorial cutting planes, such as the triangle inequalities. While the relaxation without such constraints can be solved quite efficiently for graphs with n up to 1000, the inclusion of all $O(n^3)$ triangle inequalities leads to a nontrivial computational effort even if n is rather small ($n \approx 100$). We show how to use bundle methods to deal with the triangle constraints indirectly, through Lagrangian relaxation. The resulting relaxation of Max-Cut is often very tight, and can be computed efficiently.

We provide computational results of this relaxation in combination with a Branch and Bound algorithm to get exact solutions of the underlying Max-Cut problem. Using this bounding technique we are able to solve instances not solvable by other existing methods.

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