To the Equivalence Problem of Second-Order Ordinary Differential Equations

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Abstract: S.Lie [1] showed that a second-order equation $y'' = F(x, y, y')$ is linearizable if, and only if, $\frac{\partial^4 F}{\partial y'^4} = 0$, and the coefficients of this equation satisfy two conditions $L_1 = 0$ and $L_2 = 0$. Any equation with $\frac{\partial^4 F}{\partial y'^4} = 0$ can be transformed by a change of the independent and dependent variables

$$t = \varphi(x, y), \quad u = \psi(x, y) \tag{1}$$

to the equation with $L_2 = 0$, and the relation $L_2 = 0$ is conserved with respect to (1) if, and only if, $\varphi = \varphi(x)$ [2]. One part of the presentation is devoted to the study of invariants of such transformations for equations with $\frac{\partial^4 F}{\partial y'^4} = 0$. Classes of equations which have simple values of the invariants are considered.

Another part of the presentation is devoted to the equivalence problem of the first (PI) and second Painlevé equations (PII)

$$y'' = 6y^2 + x, \quad y'' = 2y^3 + xy + \alpha.$$

Conditions which are sufficient and necessary for an equation $y'' = F(x, y, y')$ to be equivalent to PI or PII are obtained.

Since intermediate calculations in the presented above problems are cumbersome all analytical calculations were made by using symbolic computer manipulations programs. For solving the problems the series of procedures applied in [3] were used.

References:

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