

# Infinite Horizon Optimal Control Problems and their Applications

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**Abstract:** Infinite horizon optimal control problems are considered, which are of following type:

$$J(x, u) = \int_0^{\infty} f_0(t, x(t), u(t)) e^{-\rho t} dt \longrightarrow \text{Min !}$$

subject to  $x \in W_p^{1,n}(0, \infty)$ ,  $u \in L_p^r(0, \infty)$ , satisfying a. e. on  $(0, \infty)$   
*state equations*

$$x'(t) = g(t, x(t), u(t)),$$

*control restrictions*

$$u(t) \in U, \quad U \in \text{Comp}(R^r) \setminus \{\emptyset\},$$

*initial conditions*

$$x(0) = x_0$$

and *state constraints*

$$x(t) \in \overline{X(t)} \iff h_l(t, x(t)) \leq 0 \text{ for all } t \in (0, \infty), \quad l=1, \dots, w.$$

We discuss difficulties in posing and solving such problems arising because of a noncompact interval of integration and differences between Lebesgue and improper Riemann integrals. It is illustrated by use of various academical examples as well as some applications from biology and economics. Results concerning optimality conditions for the studied class of problems are presented and finally some aspects of numerical realization of integrals on unbounded intervals are considered.

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