Mathematical Modelling of Ion Transport through Membranes

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Abstract: Mathematical modelling and simulation of ion transport of ions inside and outside living cells, in their cyto-plasma and their nucleus, separated by membranes, are decisive for a better understanding of the bio-system cell. We start from Nernst-Planck equations as model equations for the ion transport and the electric potential on both sides of the membrane together with appropriate transmission conditions across the cell membrane. The membrane is perforated by channels placed in periodically distributed units (also called cell). The thickness of the membrane and the diameter of these cells are of order ϵ. Charges fixed to the channels are included modelling the influence of the channels and the changes of its conformation. Effective laws for the ion transport trough membranes are derived performing an asymptotic analysis with respect to the scale parameter ϵ. These conditions are determined solving micro-problems for cell problems in the membrane. The resulting limit equations differ from the well-known Hodgkin-Huxley equations, describing the jump of the potential at the membrane and its dynamic. Up to now, these equations are mainly phenomenological and derived from first principles. The relation between the derived new model system and the Hodgkin- Huxley equations will be discussed.

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