Submodular Function Minimization

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Abstract: A set function f defined on the subsets of a finite set V is said to be submodular if it satisfies

$$f(X) + f(Y) \geq f(X \cup Y) + f(X \cap Y), \qquad \forall X, Y \subseteq V.$$

Submodular functions are discrete analogues of convex functions. They arise in various branches of applied mathematics such as game theory, information theory, and queueing theory. Examples include the matroid rank functions, the cut capacity functions, and the entropy functions.

The importance of submodular functions in the context of combinatorial optimization was first pointed out by J. Edmonds. Most efficiently solvable combinatorial optimization problems such as the minimum spanning tree and maximum flow problems are related to submodular functions in various ways.

The first polynomial-time algorithm for minimizing submodular functions is due to Grötschel, Lovász, and Schrijver. This algorithm relies on the ellipsoid method, which is not efficient in practice. Recently, combinatorial strongly polynomial algorithms have been developed independently by Schrijver and by Iwata, Fleischer and Fujishige. Although theoretical running time bounds of these algorithms are not very attractive, one may expect to obtain more efficient algorithms for special cases that arise in applications.

In this talk, we review algorithms and applications of submodular function minimization. In particular, we mention recent works that develop new efficient algorithms for minimizing special classes of submodular functions that arise in applications such as the free energy computation of Potts models in statistical physics and the performance analysis of multiclass queueing systems.

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