New Formulation for the Ring Star Problem

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Abstract: In the Ring Star Problem(RSP) the aim is to locate a simple cycle through a subset of vertices of a graph with the objectives of minimizing the sum of two costs: a routing cost proportional to the length of the cycle, and an assignment cost from the vertices not in the cycle to their closest vertex on the cycle. The cycle must pass through some central vertex v_1. The problem has several applications in telecommunications network design and in rapid transit systems planning. It is an extension of the classical location-allocation problem introduced in the early sixties and closely related versions have been recently studied by several authors. M. Labbé, G. Laporte, I.M. Martin, J.J.S. Gonzalez ($Networks$ Vol. 43 No. 3, pp 177-189, 2004) have given a mixed integer formulation for RSP based on a partial description of the cycle polytope (P. Bauer, $Math.\ of\ OR$ Vol. 22, No. 1, pp 110-145 1997). The cycle polytope which is the convex hull of the incidence vector of all the cycles of the graph can be viewed as a relaxation of the projection of RSP on the edges of the graph. However, it's complete structure is rather very complex and remains unknown (J.F. Maurras, V.H. Nguyen, $EJOR$, Vol. 137, pp 310-325, 2002). In this paper, we propose a new formulation of RSP based on the undirected st-paths instead of the cycles in the graph. This gives a more precise formulation of RSP since the st-path polyhedron is exactly (not a relaxation) the projection of RSP on the edges of the graph and moreover we know a complete description of this polyhedron as in our case the routing costs are positive. From this formulation we derive new facets for the convex hull of the feasible solutions of RSP and we design a branch-and-cut algorithm to solve the problem. Computational results show that with same parameters of heuristics, branching, etc., our formulation permit to reduce significantly the search tree's size (of the branch-and-cut method) in compare to the formulation of Labbé et al.

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