

# Modelling in Vitro Lung Branching Morphogenesis

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**Abstract:** Lung branching morphogenesis has been widely studied in the field of developmental biology. Lung airway trees consist of relatively regular-sized distal branches, but how this regular branch pattern is formed is not well understood. In the present study, we construct a mathematical model with a free boundary that can reproduce the dynamics of *in vitro* branching morphogenesis. The model is confirmed by numerical simulations.

A detailed analysis of the model is presented: we prove the existence of travelling waves and investigate their stability with respect to 2-dimensional periodic perturbations. Further, we consider via asymptotic analysis two extreme cases: large and small kinetics. For large kinetics the models reduce to the well-studied Stefan problem, which is known to exhibit a diffusive-instability and hence shows branch patterns. Considering small kinetics, small perturbations are smoothed out resulting in regular shapes of the cultures. The branching phenomenon in general can be understood as somewhere between these two extremes.

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