Combined Finite-Volume and Local Level-Set Approach on Unstructured Grids

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Abstract: A multi-dimensional front tracking concept is presented for solving flow problems with embedded discontinuous solution features. The tracking method is based on a level-set approach with a restricted dynamic definition range on a narrow band in the vicinity of the fronts. The approach is superposed to unstructured grids of arbitrary cell structure as used for finite-volume computations.
The transport step $\frac{\partial G}{\partial t} + \vec{c} \cdot \nabla G = 0$ of the level-set function $G(\vec{r}, t)$ is solved by an explicit scheme up to second order accuracy in time. The gradients ∇G are updated on unstructured grids by a least square method. The determination of the front velocity $\vec{c}$ differs between material interfaces, where $\vec{c}$ is the fluid velocity defined uniquely around a front, and between gas dynamic waves, as shock waves. Here, the front velocity $\vec{c}$ is defined on the front location only and is a function of the jump conditions left and right the front. Thus the derivation of $\vec{c}$ and the distribution over the definition range of G requires special procedures. The level-set function G is normalized (re-initialized) for reasons of accuracy to a distance function $|\nabla G| = 1$. The gradients and the new, normalized level-set function are determined by pointwise least square approaches on arbitrary grid structures. This normalization procedure performs iteratively (index k) in two steps starting from an initial field of values $G^{k=0}$. The direction of the gradient ∇G^k at a node near the front is computed in the first step and the functional value of G^k is set to satisfy the slope condition $|\nabla G| = 1$ in the second least square step. The normalization has shown to be the most expensive part of the tracking method in the case of complex front contours.
The level-set front tracking method is coupled with a node-centered finite-volume method "Mouse", developed at the site of the Institute. The coupling, in particular the flux evaluation, is realized by a double sided flux formulation at computational cells cut by a front what is called here flux-separation. The double sided fluxes are defined either by MUSCL extrapolation or by "inner boundary conditions" given by the jump conditions over the discontinuity.
The combined finite-volume and tracking method and a number of aspects related to the tracking of ending or crossing fronts will be discussed in more detail. The flexibility of the tracking method is demonstrated by a number of different flow problems with discontinuities as they are e.g. 2-D or 3-D shock waves, detonations, shock-bubble interaction or free surface flows around ships.

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