Bidimensional Sand Pile Model

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Abstract: In this paper we introduce the Bidimensional Sand Pile Model ($BSPM$), that is, a generalization of the Sand Pile Model (SPM) with the addition of a further dimension. The SPM and some related models have been studied in many different domains and they play an important role in the theory of discrete dynamical systems. They were considered in the context of integer lattices by Brylawski in 1973. From a physics point of view, Bak, Tang, and Wiesenfeld used them in order to illustrate the important notion of *self organisation criticality*. Moreover, Anderson et al., Spencer, and Goles and Kiwi studied them from a combinatorial point of view.

SPM is a discrete dynamical system describing pilings of granular objects distributed on an array of columns. In 1993, Goles and Kiwi introduced this model in and proved that SPM has a unique *fixed point*, i.e. a state in which no sand grain can fall under the evolution rule. Moreover, they showed that the order induced by SPM on accessible partitions is a suborder of the dominance order, introduced by Brylawski. Later, in 1998, Goles, Morvan, and Phan considered a generalisation of the SPM: the Ice Pile Model (IPM). After that, Latapy, Mantaci, Morvan, and Phan extended SPM to SPM(∞), a natural extension of SPM when one starts with an infinite number of grains. By using two different approaches they gave recursive formulae for $|SPM(n)|$.

In this paper we define an extension of the Sand Pile Model SPM and more generally of Ice Pile Model IPM by adding a further dimension. In order to do it, we extend the previous rules so that grains can fall and slide to the east and to the south, and so that the configurations obtained are plane partitions. By drawing a parallel between these unidimensional and bidimensional models we will find some common features and some differences. We will show that, like for SPM, not all plane partitions are accessible in $BSPM$ starting from the initial state. However, it appears to be much more difficult to characterize the partitions that are accessible in $BSPM$: we will five some necessary but not sufficient conditions for a partition to be accessible. At the end, we give some results on the fixed point problem of this model.

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