

# Dirichlet Feedback Control for the Stabilization of Unstable Parabolic Systems: Application to Heat Transfer Control in Fluid Flow

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**Abstract:** We consider the stabilization problem by means of Dirichlet feedback control for systems modeled by scalar or vector parabolic equations. The main emphasize is put on the treatment of nonlinear reaction-advection-diffusion type equations which are unstable if uncontrolled. Two different numerical approaches are analyzed and compared. The first method which was first presented by A. V. Fursikov relies on the reformulation of the stabilization problem in an extended geometrical domain taking into account the stability properties of the related eigenfunctions. The second approach is based on the formulation of the stabilization problem as an optimal control problem eventually including feedback control. In that context a new numerical method which relies on DG-FEM (Discontinuous-Galerkin) for the time discretization as well as continuous finite elements for the space discretization is presented. A modified BFGS scheme combined with a new checkpointing strategy for the computation of the adjoint problem lead to an efficient scheme. Applications to heat transfer control in fluid flow are presented.

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