Implementation of the Branch-and-Bound Method for Non-Convex Quadratic Optimization Problems Using the Successive Convex Relaxation

C. C. Vo[1], **A. Takeda**[2], **K. Fujisawa**[3], **and M. Kojima**[1]

Abstract: The general Quadratic Optimization Problem (QOP) has a linear objective function and a feasible region described by linear and quadratic constraints. Difficulities in solving the QOP come from its nonconvexity. This paper deals with feasibility and accuracy of approximate solutions of the QOP.

We can bound optimal values of the QOP using the Successive Convex Relaxation (SCR) method. It is well known that the speed of the SCR method, feasibility and accuracy of approximate solutions depend on how large and on how much nonconvex the feasible region is. We propose a Branch-and-Bound (BnB) algorithm in which the original QOP is partitioned into subproblems whose feasible regions are subsets of the original one. The branching operations are designed to reduce nonconvexity of the subproblems. As the nonconvexity of the subproblems decreases and the diameter of the feasible regions becomes smaller, the algorithm generates an almost feasible $\varepsilon-$optimal solution of the given QOP in finite computational time.

To solve large scale QOPs, the algorithm was implemented in a highly parallel fashion. Semi-Definite Problems (SDP) involved from the SCR method were solved using the software SDPA that was run in parallel on a local network.

[1] Department of Mathematical and Computer Sciences,
Tokyo Institute of Technology
vo.chi.cong@is.titech.ac.jp,kojima@is.titech.ac.jp

[2] Toshiba Corporation
akiko.takeda@toshiba.co.jp

[3] Department of Mathematical Sciences,
Tokyo Denki University
fujisawa@rnc.r.dendai.ac.jp