Efficient Synchronization Algorithms for 1-Bit Inter-Cell Communication Cellular Automata

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Abstract: In recent years cellular automata (CA) have been establishing increasing interests in the study of modeling concurrent phenomena occurring in biology, chemistry, ecology, economy, geology, mechanical engineering, medicine, physics, sociology, public traffic, etc. Cellular automata are considered to be a good model of complex systems in which an infinite one-dimensional array of finite state machines (cells) updates itself in synchronous manner according to a uniform local rule.

In this paper, we study a firing squad synchronization problem for a large scale of cellular automata (CA) on one- and two-dimensional 1-bit inter-cell communication cellular automaton (CA_{1-bit}). A 1-bit inter-cell communication model studied in this paper is a new CA whose inter-cell communication is restricted to 1-bit. We call the model 1-bit CA in short. The number of internal states of the 1-bit CA is assumed to be finite in a usual way. The next state of each cell is determined by the present state of itself and two binary 1-bit inputs from its left and right neighbor cells. Thus the 1-bit CA can be thought to be one of the most powerless and simplest models in a variety of CAs.

It is shown that there exists a CA_{1-bit} that can synchronize 1-D n cells with the general on the kth cell in $n + \max(k, n - k + 1)$ steps, which is two steps larger than the optimum one that was developed for O(1)-bit communication model. In addition, we give a twodimensional CA_{1-bit} which can synchronize any $n \times n$ square and $m \times n$ rectangular arrays in 2n - 1 and $m + n + \max(m, n)$ steps, respectively. Lastly, we propose a generalized synchronization algorithm that operates in linear steps on two-dimensional rectangular arrays with the general located at an arbitrary position of the array. The time complexities for the first three algorithms developed are one to four steps larger than optimum ones proposed for O(1)-bit communication model. We show that there still exist several new synchronization algorithms, although more than 40 years have passed since the development of the problem.

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