Relaxations for Robust Linear Matrix Inequalities

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Abstract: A robust linear matrix inequality (RLMI) is given by

$$F_0(p) + \sum_{i=1}^{m} F_i(p)\lambda_i \geq 0 \ , \ \forall p \in H \tag{1}$$

where $p \in \mathbb{R}^n$ is the parameter vector, $\lambda_1, \ldots, \lambda_m$ are the decision variables, and $F_0(p), F_1(p), \ldots, F_m(p)$ are real symmetric matrices which depend linearly on the parameter vector p. The set H is assumed to be a convex subset of $\mathbb{R}^n$ which is often assumed to be a hyperrectangle or more general a convex polytope. To compute an appropriate set of decision variables such that (1) is fulfilled is the goal of recent research in the control theory community. A number of important problems in automatic control have been reformulated as RLMI and to find effective solution methods is of great importance for real applications. A typical example is the problem of deciding if a matrix with uncertain entries has all eigenvalues in the open left half plane of the complex plane which corresponds to robust asymptotic stability of the corresponding linear system. If the set H is a hyperrectangle it is known that the RLMI (1) can be reformulated as a set of ordinary linear matrix inequalities (LMI) which can be solved with very efficient interior point methods. The main drawback of this reformulation is that the number of ordinary LMI depends in general exponentially on n which limits the application to real world problems. Thus, it is usefull to look for relaxations of (1) which avoid this exponential complexity and lead to good solutions for practical problems. In the literature some ad hoc approaches for the construction of such relaxations have been published. In this contribution we will use some new results from real algebraic geometry, namely, representations of positive polynomials on semialgebraic sets, which will enable us to compute a sequence of relaxations which approximate (1). For several examples from the literature we will show that our new relaxations are more efficient than the previously known, and we will show the results of intensive numerical studies.

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