Quasi-Optimization Problems: The Existence of Solutions and Solving Methods

N. X. Tan[1]

Abstract: Let X, Y be topological vector spaces, D, K be subsets of X and Y, respectively. Let $S : D \to 2^D$ and $T : D \to 2^K$ be multivalued mappings with $S(x), T(x)$ nonempty for all $(x, y) \in D \times K, F : D \times K \times D \to R$ be a real function. We are interested in finding a point $(\bar{x}, \bar{y}) \in D \times K$ such that

1) $\bar{x} \in S(\bar{x})$;

2) $\bar{y} \in T(\bar{x})$;

3) $F(\bar{x}, \bar{y}, \bar{x}) = \min\limits_{x \in S(\bar{x})} F(\bar{x}, \bar{y}, x)$.

This problem is called a quasi-optimization problem and $(\bar{x}, \bar{y})$ is said to be its solution. The propose of this talk is to show some sufficient conditions on the existence of solutions for this problem and some relations to some other problems in the optimization theory. This problem can be also formulated for the vector case and has many relations with different vector problems. To solving methods we can use the fixed point algorithm . But, this algorithm works very slowly and with low dimensions. In this talk we construct a sequence (x_n, y_n) with

a) $x_{n+1} \in S(x_n)$;

b) $y_n \in T(x_n)$;

c) $F(x_n, y_n, x_{n+1}) = \min\limits_{x \in S(x_n)} F(x_n, y_n, x)$

and find some conditions on the mappings S, T and F such that this sequence converges to a solution of the above problem.

[1] Institute of Mathematics
National Center for Sciences and Technology
P. O. Box 631 - Bo Ho, Hanoi, Vietnam
nxtan@thevinh.ncst.ac.vn