Semi-smooth Newton and Augmented Lagrangian Methods for a Simplified Friction Problem

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Abstract: In this talk a simplified friction problem and algorithms for its solution are presented and analyzed in an infinite-dimensional function space framework. While typically first order methods for the solution of these problems were used, we propose second order methods that possess additional advantages such as termination after a finite number of iterations. The problem can be stated as minimization of the non-differentiable functional

$$J(y) := \frac{1}{2}||\nabla y||^2_{L^2(\Omega)} + \frac{\mu}{2}||y||^2_{L^2(\Omega)} - (f, y)_{L^2(\Omega)} + g \int_{\Gamma_f} |\tau y(x)|\, dx$$

over the set

$$Y = \{y \in H^1(\Omega) : \tau y = 0 \text{ a.e. on } \Gamma_0\},$$

where $\Omega \subset \mathbb{R}^n$ is open and bounded, $g > 0, \mu \geq 0$, $\partial\Omega =: \Gamma_0 \dot\cup \Gamma_f$, $f \in L^2(\Gamma_f)$ and τ denotes the trace operator. The dual to the above problem turns out to be a constrained maximization of a smooth functional. Based on this dual formulation a regularization procedure is chosen that allows to state and investigate our algorithms at the continuous level. We present a primal-dual active set strategy and a semi-smooth Newton method for the regularized problem as well as an augmented Lagrangian method for the original dual problem and analyze their close relationship. Local as well as global convergence results for the algorithms are given. By means of numerical tests we discuss among others the dependence on the mesh, the role of the regularization and the number of required iterations, compare the various approaches and illustrate the efficiency of the proposed algorithms for the static case given above as well as for a dynamical version of the simplified friction problem.

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