The $B = 0$ Constraint vs. Minimization of Numerical Errors in MHD Simulations

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Abstract: The MHD equations are a system of non-strictly hyperbolic conservation laws. The non-convexity of the inviscid flux vector resulted in corresponding Jacobian matrices with undesirable properties. On the other hand, the MHD equations can be derived from basic principles in either conservative or non-conservative form. The non-conservative system has a better conditioned eigensystem. The $\mathbf{B} = 0$ constraint of the MHD equations is only an initial condition constraint. One does not need the $\mathbf{B}$ condition to close the MHD system. We formulate our new low dissipative high order scheme together with the Cargo & Gallice (1997) form of the MHD approximate Riemann solver in curvilinear grids for both versions of the MHD equations. A novel feature of our new method is that the well-conditioned eigen-decomposition of the non-conservative MHD equations is used to solve the conservative equations. This new feature of the method provides well-conditioned eigenvectors for the conservative formulation, so that correct wave speeds for discontinuities are assured. The justification for using the non-conservative eigen-decomposition to solve the conservative equations is that our scheme has a better control of the numerical error associated with the $\mathbf{B}$ condition. Consequently, computing both forms of the equations with the same eigen-decomposition is almost equivalent. It will be shown that this approach, using the non-conservative eigensystem when solving the conservative equations, also works well in the context of standard shock- capturing schemes.

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