An Approach to Parameter Estimation and Model Selection in Differential Equations

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Abstract: The estimation problem in parameterized systems of differential equations starts with data acquired through observations of system trajectories made in the presence of noise and seeks to estimate the parameter values by solving an optimization problem which matches solutions of the differential equation to the observed data. Thus there is an explicit stochastic component to the problem. Typically, two classes of method are considered:

1 Explicitly computed solution trajectories are compared directly with the observations in an unconstrained optimization procedure; and

2 the system of differential equations is imposed as explicit constraints on the optimization problem and the resulting mathematical program typically is solved by a variant of sequential quadratic programming.

When the model is known then the Gauss-Newton or scoring method appears the method of choice for the first class of methods, and there are good reasons for this which are a consequence of the stochastic setting. Similar approximations appear to work well for the second class of methods, and experience here will be summarized.

It all becomes harder if the only information available is that the model is known to lie within a parameterized class of systems. Presumably one should start the searching with the simpler members of this class (the potentially underspecified systems). However, the scoring method looses its justification in this case. A procedure which offers the possibility of overcoming this difficulty is being studied for systems linear in the state variables. Key components include the use of a multiple shooting version of the Kalman filter for problems with nontrivial dichotomy, and the use of information criteria for discriminating between contending models.

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