

ML(k)BiCGSTAB Algorithm with Adaptive Determination of k for Solving Sparse Linear Systems

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Abstract: We are principally concerned with the solution of large sparse systems of linear equations

$$Ax = b \quad (1)$$

using a variant of the popular BiCGSTAB algorithm. In 1992, van der Vorst proposed the BiCGSTAB algorithm. This algorithm is derived from the BiCG scheme which is a Lanczos-based Krylov subspace algorithm and it does not seem to suffer from an irregular convergence behavior in comparison with CGS algorithm. In 1993, Gutknecht and Sleijpen and Fokkema derived from BiCGSTAB algorithm to the variants which called BiCGSTAB2 and BiCGSTAB(ℓ), respectively. The purpose of doing so was to increase the robustness and speed up of the convergence of BiCGSTAB algorithm [1]. Recently, Yeung and Chen [2] has been constructed the another variant of the BiCGSTAB algorithm, which is called ML(k)BiCGSTAB. This algorithm based on the Lanczos process. Namely, the underling idea is adaptively use multiple left starting Lanczos vectors. However, the ML(k)BiCGSTAB algorithm has a serious problem how to determine k . Nowadays, we can derive and propose an efficient implementation of determining k , using residual smoothing techniques for iterative algorithms. This is referred to as the adaptive minimal residual smoothing.

The practical performance of this technique is illustrated in a lots of numerical experiments. Comparing the other BiCGSTAB algorithms, the numerical experiments show the efficiency and robustness of the ML(k)BiCGSTAB algorithm with adaptive determination of k by using minimal residual smoothing. At last, we will give a summary of the work and future work directions.

References

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