Spatio-Temporal Patterns in far from Equilibrium States from the Viewpoints of Chemical and Bioligical Systems

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Abstract: Reaction-diffusion equations have been often used as continuous model to describe patterns and waves arising in far from equilibrium states. In spite that the equations look so simple, numerics has revealed that the equations generate unexpectedly complex but regulated spatio-temporal patterns. One of the typical equations is the following autocatalytic reaction diffusion equations for two components u and v:

$$\begin{aligned} u_t &= d_u \Delta u + uv^m - au - bu^n \\ v_t &= d_v \Delta v - uv^m - a(1-v) \end{aligned} \tag{1-1}$$

where a and b are positive constants. In particular, when $m = 2$ and $n = 1$, (1-1) is called the Gray-Scott equations([1], for instance). For $m = 2$, it is numerically known that (1-1) generates diverse complex patterns in higher dimensions, depending on values of m and n. One typical phenomenon is occurrence of spot patterns which are generated through self-replicating process. Our understanding of the reason why such complex patterns occur in (1-1) is that it possesses feeding process, or an open system. On the other hand, in the absence of such process in (1-1) with $a = 0$, (1-1) reduces to the following closed systems

$$\begin{aligned} u_t &= d_u \Delta u + uv^m - bu^n \\ v_t &= d_v \Delta v - uv^m \end{aligned} \tag{1-2}$$

In a bounded domain with the Neumann boundary conditions, it is proved that any solution (u, v) of (1-2) becomes spatially homogeneous asymptotically, that is there occurs no pattern formation at all. There fore, we have believed for long tome that such closed systems are not so interesting from pattern formation viewpoints. However, quite recently, we have found that (1-2) generate a very complex patterns in transient process, although it generate no patterns asymptotically. For the study of such transient behavior, I would like to discuss a class of reaction-diffusion systems from analytical and complementarily numerical methods in this conference

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