## **Higher Order Repetitive Control**

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**Abstract:** Repetitive control (RC) applies to situation in which a control system executes a periodic command, and it aims for zero tracking error by examining the error in the previous period. Applications include positioning in computer disk drives, vibration cancellation, machining, rolling operations for production of sheets of material, etc. Iterative learning control (ILC) is a closely related field where a control system is given the same command repeatedly, but each time it returns to the same initial condition before the next command is given. The ILC literature has many contributions developing what are called higher order ILC laws, where the control action in the current repetition is a function of the errors observed in multiple previous runs. This paper develops the analogous higher order control for repetitive control applications, making the repetitive control updates a function of the error in more than just the last period of the command.

First, a stability theory is developed for higher order RC. The stability boundary is developed based on a Nyquist approach. In both RC and ILC there is a sufficient condition for stability based on frequency response that is particularly useful in design. This paper establishes how this condition can be generalized to higher order RC. It is established that the resulting condition is again sufficient for stability, and also that for nearly all applications it is extremely close to the actual boundary.

In order to evaluate the potential benefits of higher order RC, studies are made to examine the convergence rate and the final value of the tracking error reached by using first order RC and by going to higher order RC. By averaging over more than one period in the error history, one decreases the effect of noise on the final error level. It is seen that one also decreases the speed of convergence. First order RC can also decrease the effects of noise by using a smaller gain. Hence, the paper studies the trade off between these two approaches to RC design.

Besides learning rate and final value of the error from broadband noise, there is one other basic issue in evaluating the performance of higher order RC. This is the performance in the presence of periodic errors at frequencies other than those in the command. The watered effect is a fundamental limitation in all real time feedback control. By going to higher order, the error frequency components that are significantly amplified by the repetitive control action lie closer to the addressed frequencies, and the notches for the addressed frequencies are made narrower. This suggests that higher order RC can give improved performance when the addressed frequency is known accurately and there are other significant disturbances at separated frequencies.

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