Robust Parameter Estimation in Systems of Differential Equations

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Abstract: The problem of identification of unknown parameters in dynamic models is among the most important tasks in mathematical modeling of dynamic processes. The identification problem can be described as follows. Let the dynamics of the model be described by a system of ordinary differential equations $\dot{x} = f(t, x, p)$, where the right-hand side f depends on an unknown vector of parameters p. It is assumed that there is a possibility to measure a signal η of an output device that writes at given time points $t_j,\ j = 1, .., k$, the output signal $q(t_j, x(t_j), p)$, of the dynamic system with some error $\varepsilon(t_j) : \eta(t_j) = q(t_j, x(t_j), p) + \varepsilon(t_j),\ j = 1, ..., k$. According to the common approach, in order to determine the unknown parameters the optimization problem is solved in which the special functional is minimized under constraints that describe the specifics of the model. Any norm of the measurement error may be used as the functional in the optimization problem. The type of the norm may be determined by the statistical distribution of the measurement error. If the errors are independent, normally distributed with zero mean and known variances σ_j^2, minimizing a weighted least squares function

$$\min_p \sum_j (\eta(t_j) - q(t_j, x(t_j), p))^2 / \sigma_j^2$$

yields a *maximum likelihood estimate*. But in case of Laplace - distribution l_1 estimation is appropriate:

$$\min_p \sum_j |\eta(t_j) - q(t_j, x(t_j), p)| / |w_j|$$

Further, it is well-known that l_1 parameter estimation possesses the property of robustness, that is the optimal solution is *insensitive to the effect of outliers* in data.

The paper focuses on the method for robust parameter estimation for dynamic systems which combines the boundary value problem approach and a special method of solving l_1 approximation problems. The method is successfully applied to several real-life problems.

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