

Helical Waves Arising in Some Reaction-Diffusion Systems

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Abstract: In this talk, we show that helical waves can bifurcate directly from planar traveling waves by using a mathematical model.

A helical wave is observed in self-propagating high-temperature syntheses (SHS), for instance. One can create a high-quality uniform product by the SHS when a combustion wave keeps its profile and propagates at a constant velocity, that is, combustion waves of steady-state mode (planar traveling waves) bring high quality products. When experimental conditions are changed, however, the planar traveling wave may lose its stability and give place some non-uniform ones. Actually, a planar pulsating wave appears through the Hopf bifurcation of planar traveling wave. Moreover, we observe a wave that propagates in the form of spiral encircling the cylindrical sample with several reaction spots. In our talk, this wave is called a helical wave since it has been shown by our 3D numerical simulation that the isothermal surface of the wave has some wings and it helically rotates down as time passes on. It is observed that the number of wings is the same as that of reaction spots on the cylindrical surface. Similar helical waves are observed also in propagation fronts of polymerizations in laboratory and they are obtained also by numerical simulation of some autocatalytic reactions as well as the SHS.

We have been interested in the existing condition of stable helical wave and the transition process of wave patterns from steady-state mode to pulsating mode and/or helical mode. In this talk we report the following mathematically rigorous results obtained by using a reaction-diffusion system, which is very simple model system but seems to be essentially similar to those for the SHS and the autocatalytic reaction:

- (1) A stable helical wave with several reaction spots can appear directly by the Hopf bifurcation of planar traveling wave.
- (2) Even if a traveling wave is stable in the one-dimensional problem, the corresponding planar traveling wave can be unstable in the two- or three-dimensional problem and a helical wave takes the place of planar traveling wave.

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