Divergence Cleaning and Absorbing Boundary Conditions for the MHD Equations

A. Dedner[1] and D. Kröner[1]

Abstract: For the simulation of magnetic flux tubes in the atmosphere of the sun we use the mathematical model consisting of the ideal compressible equations of magnetohydrodynamics (MHD), a nonlinear hyperbolic system, which can be derived from the Euler equations of gas dynamics and the Maxwell equations. The MHD equations describe the complex couplings between the flow variables, i.e. the density, the velocity, the total energy, the pressure tensor, the gravitational force with the magnetic field. We have to consider the system in the atmosphere of the sun, i.e. an unbounded domain in 3D. For simplicity we have to restrict ourselves to a finite domain of computations.

But this implies the necessity of the definition of artificial boundary conditions. They are not given by physical reasons but they have to be chosen such that they guarantee that the solution on the bounded computational domain is a good approximation to the exact solution of the original problem on the unbounded domain.

Another main problem which is related to the MHD equations concerns the divergence free magnetic field. Usually the initial data for the magnetic field are divergence free and the exact solution will keep this property for all times. But due to numerical errors the numerical magnetic field will loose this property for later times. For the discretization of the divergence-free property of the magnetic field many attempts can be found in the literature. In this lecture we will discuss a new method which generalizes previous ideas of Munz and Sonnendruecker to the MHD equation. Roughly speaking the system is extended and perturbations of the divergence of the magnetic field will be transported away to the boundary.

These results are based on joint papers with A. Dedner, F. Kemm, C.-D.Munz, M. Wesenberg, T. Schnitzer, and I. Sofronov.

[1] Abteilung Angewandte Mathematik
Universitaet Freiburg
Hermann-Herder-Str.10
79104 Freiburg
Germany
dietmar@mathematik.uni-freiburg.de