Parallel Decomposition Methods for Large-Scale Constrained Variational Problems

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Abstract: A common strategy to approach complex, convexly constrained variational (variational inequality, minimization, feasibility) problems is to decompose their feasibility set S into a possibly infinite intersection of simple convex sets $(S_i)_{i\in I}$ and to solve a sequence of intermediate problems over these, in such a way that the intermediate solutions converge to a solution of the original problem. The resulting algorithm is block-iterative and parallel if the passage from iteration n to iteration $n+1$ involves a block of concurrently activated constraint sets $(S_i)_{i\in I_n\subset I}$. From a numerical standpoint, such methods are very attractive due to the great flexibility they offer in terms of implementation on parallel processing architectures.

The main point of this talk is that a number of apparently unrelated block-iterative parallel algorithms employed in convexly constrained variational problems share common structures and that their convergence is governed by common principles. Such structures and principles will be brought out and studied in a Banach space setting. This will allow us not only to recast and analyze existing algorithms in a simplified and unified framework, but also to devise extensions of these algorithms within this framework. Applications to variational inequalities, minimization, and feasibility problems will be discussed. The efficiency of the methods will be illustrated through numerical applications to large-scale signal and image recovery problems.

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